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Holographic Duals of Kaluza-Klein Black Holes

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Abstract

We apply Brown-Henneaux's method to the 5D extremal rotating Kaluza-Klein black holes essentially following the calculation of the Kerr/CFT correspondence, which is not based on supersymmetry nor string theory. We find that there are two completely different Virasoro algebras that can be obtained as the asymptotic symmetry algebras according to appropriate boundary conditions. The microscopic entropies are calculated by using the Cardy formula for both boundary conditions and they perfectly agree with the Bekenstein-Hawking entropy. The rotating Kaluza-Klein black holes contain a 4D dyonic Reissner-Nordström black hole and Myers-Perry black hole. Since the D-brane configurations corresponding to these black holes are known, we expect that our analysis will shed some light on deeper understanding of chiral CFT_2's dual to extremal black holes.

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Contents

1 Introduction

More than ten years have passed since the Bekenstein-Hawking entropy was calculated microscopically by using string theory [1]. The essential strategy for microscopic entropy counting is to construct a black hole geometry by D-branes and then to map the problem of the black hole to a dual boundary field theory on the D-branes. Then by using the BPS properties, we can count the microscopic entropy of the black hole in the weak coupling field theory.

After this accomplishment, this strategy is applied to various kinds of black holes and even other black objects like black rings. In addition to this, the AdS/CFT correspondence, a kind of duality between gravity and field theory, inherits this feature and refines or makes this strategy more stringent in certain setups [2–4].

In the context of AdS/CFT, black hole microstates are investigated mainly by using AdS_3/CFT_2 correspondence since AdS_3 appears as a near-horizon geometry of 5D extremal black holes and CFT_2 is well analyzed. When we consider 4D extremal black

holes, on the other hand, AdS_2 naturally appears as a near-horizon geometry. However, AdS_2/CFT_1 is still mysterious because we do not know CFT_1 well. For some context, CFT_1 is regarded as conformal quantum mechanics (CQM) [5–11], while it is as 2D chiral conformal field theory [12–15]. In this paper we investigate the latter.

Going back to the history of the relation between conformal field theory and gravity, in 1986, Brown and Henneaux succeeded in identifying generators of Virasoro algebras living on the boundary of AdS_3 spacetimes with the diffeomorphisms preserving a certain boundary condition [16]. They found that the central charge of the Virasoro algebra is related to the AdS_3 radius. This method is applied to microscopically derive the entropy of the BTZ black hole [17]. Subsequently, related methods were developed [18–21], and in this direction many works were done (for example, [22–25]).

Recently, the entropy of the 4D extremal Kerr black hole was microscopically calculated by applying Brown-Henneaux's method to the near-horizon geometry of it [26]. This is interesting partly because their calculation was not based on supersymmetry nor string theory. However, the Brown-Henneaux's method does not tell us what is the boundary theory, thus it is desired to find the boundary theory explicitly. Microscopic counting of neutral black holes like Kerr black hole, is very difficult in that we do not know how to realize this neutrally charged geometry by D-branes. In [27], the entropy of the 4D extremal Kerr black hole was microscopically calculated by relating it to a rotating Kaluza-Klein (KK) black hole solution formulated in [28–31]. This solution itself is meaningful in that it includes various classes of black holes like the 4D dyonic Reissner-Nordström black hole. Since the rotating KK black holes are constructed as a D0-D6 bound state [32], their entropies are calculable microscopically from the corresponding brane configurations [33,34].

In this paper, we would like to apply Brown-Henneaux's method to the 5D extremal rotating KK black holes and calculate its entropy microscopically. We essentially follow the calculation of [26]. First we take near-horizon limit for the extremal rotating KK black holes. For this near-horizon geometry, it is shown that two completely different Virasoro algebras can be obtained as the asymptotic symmetry algebras, according to appropriate boundary conditions. These algebras will act on the Hilbert states of the holographic duals of the extremal rotating KK black hole. Although these dual boundary theories are not specified explicitly, the microscopic entropies are calculated using the Cardy formula for both boundary conditions and they perfectly agree with the Bekenstein-Hawking entropy. Therefore we expect that the two boundary conditions correspond to two different consistent holographic duals. Since the D-brane configurations corresponding to these black holes are known, we expect that our analysis will shed some light on deeper understanding of chiral CFT_2's dual to extremal black holes.

The organization of this paper is as follows. In section §2, we review the work [26]. In section §3, we review the rotating KK black holes. In section §4 we take the near-horizon limit of the extremal rotating KK black holes. In section §5, we impose boundary conditions in the asymptotic region and determine the diffeomorphisms which preserve the boundary conditions. We construct Virasoro generators in two ways since the rotating KK black holes contain two $U(1)$ fibers. Then we find the central charges of these Virasoro algebras living on the boundary. In section §6, we relate the physical parameters of the rotating KK black hole to the effective temperatures of the dual chiral CFT_2's and then, in section §7, we apply the Cardy formula to calculate the entropy microscopically. Finally, in §8, we summarize our conclusion and give some discussion.

2 A Brief Review of Kerr/CFT Correspondence

In [26], it is proposed that the 4D extremal Kerr black hole is dual to a chiral CFT_2. In the extremal limit [35], the near-horizon geometry is of the form

$$ds^2 = 2G_4 J\Omega^2\Big(-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2} + d\theta^2 + \Lambda^2(d\varphi + r d\tau)^2\Big), \tag{2.1}$$

$$\Omega^2 = \frac{1+\cos^2\theta}{2}, \quad \Lambda = \frac{2\sin\theta}{1+\cos^2\theta}, \tag{2.2}$$

where J is the angular momentum and G_4 is the 4D Newtonian constant. This angular momentum is related to the ADM mass M of the black hole as $J = G_4 M^2$ in the extremal limit. For a fixed θ, this geometry is the same as a quotient of a warped AdS_3, which is analyzed in the context of topological massive gravity in 3D [36].

A generator of the Virasoro algebra of the chiral CFT_2 is identified with a class of diffeomorphism which preserves an appropriate boundary condition on the near-horizon geometry. Then it is found that a nontrivial part of the diffeomorphism, or the asymptotic symmetry group (ASG), contains diffeomorphisms of the form

$$\zeta_n = -e^{-in\varphi}\partial_\varphi - inre^{-in\varphi}\partial_r \quad (n = 0, \pm1, \pm2, \cdots). \tag{2.3}$$

We notice that ζ_n contains ∂_φ, not ∂_τ.

The diffeomorphisms ζ_n generate a Virasoro algebra without a central charge

$$i[\zeta_m, \zeta_n] = (m-n)\zeta_{m+n}. \tag{2.4}$$

By following the covariant formalism of the ASG [37–46], a conserved charge Q_ζ associated with an element ζ is defined by

$$Q_\zeta = \frac{1}{8\pi G_d}\int_{\partial\Sigma} k_\zeta[h, \bar{g}], \tag{2.5}$$

where $\partial\Sigma$ is a spatial surface at infinity and

$$k_\zeta[h,\bar{g}] = \frac{1}{2}\Big[\zeta_\nu D_\mu h - \zeta_\nu D^\sigma h_{\mu\sigma} + \zeta^\sigma D_\nu h_{\mu\sigma} + \frac{1}{2} h D_\nu \zeta_\mu - h_{\nu\sigma} D^\sigma \zeta_\mu + \frac{1}{2} h_{\nu\sigma}(D_\mu \zeta^\sigma + D^\sigma \zeta_\mu)\Big] *_d (dx^\mu \wedge dx^\nu). \tag{2.6}$$

Here d is the spacetime dimension ($d = 4$ in the current case), $*_d$ represents the Hodge dual, $\bar{g}_{\mu\nu}$ is the metric of the background geometry (2.1) and $h_{\mu\nu}$ is deviation from it. We also notice that the covariant derivative in (2.6) is defined by using $\bar{g}_{\mu\nu}$. In addition to a charge Q_{ζ_n} associated with ζ_n, there exists a charge Q_{∂_τ} associated with ∂_τ. Since it measures deviation from the extremality, it is fixed to zero in this case.

Then let us consider the Dirac bracket of Q_{ζ_n} under the constraint $Q_{\partial_\tau} = 0$. It is determined by considering transformation property of the charge Q_{ζ_n} under a diffeomorphism generated by ζ_m. It then follows that

$$\{Q_{\zeta_m}, Q_{\zeta_n}\}_{Dirac} = Q_{[\zeta_m,\zeta_n]} + \frac{1}{8\pi G_4}\int_{\partial\Sigma} k_{\zeta_m}[\mathcal{L}_{\zeta_n}\bar{g},\bar{g}]. \tag{2.7}$$

By redefining the charge as $\hbar L_n = Q_{\zeta_n} + 3J\delta_{n,0}/2$ and replacing the Dirac bracket $\{.,.\}$ by a commutator $-\frac{i}{\hbar}[.,.]$, we see that L_n satisfies a Virasoro algebra

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}, \tag{2.8}$$

with the central charge $c = 12J/\hbar$.

The temperature of the dual chiral CFT_2 is, on the other hand, determined by identifying quantum numbers in the near horizon geometry with those in the original geometry. This method gives the temperature

$$T = \frac{1}{2\pi}. \tag{2.9}$$

From this, entropy of the 4D Kerr black hole is microscopically calculated via the Cardy formula as

$$S_{micro} = \frac{\pi^2}{3}cT = \frac{2\pi J}{\hbar}, \tag{2.10}$$

which agrees with the Bekenstein-Hawking entropy calculated macroscopically.

We note that this dual theory is not like CFT in the usual AdS/CFT correspondence, because ϕ is space-like coordinate and the Virasoro algebra does not contain the time translation generator. Moreover, the isometry of the near horizon geometry does not contain $SL(2,R)$ of the Virasoro algebra. We will see that the Virasoro algebras of the holographic duals of the rotating KK black holes also have this properties.

3 The Rotating Kaluza-Klein Black Holes

In this section we review the rotating Kaluza-Klein black holes [28–31]. This is the 5D-uplifted solution of rotating black holes with both electric and magnetic charges in the 4D Einstein-Maxwell-dilaton theory. This solution includes the dyonic ($P = Q$) Reissner-Nordström black hole in the 4D Einstein-Maxwell theory and the 5D Myers-Perry black hole, as special cases.[4] In terms of string theory, it can be interpreted as a rotating D0-D6 bound state.

We consider the 4D Einstein-Maxwell-dilaton action

$$S = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g}\Big[\mathcal{R} - 2g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{4}e^{-2\sqrt{3}\Phi}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}\Big]. \tag{3.11}$$

This theory is obtained by a usual Kaluza-Klein reduction of the 5D pure Einstein gravity theory, which is easier to deal with in many cases. In this paper, we will always work on the 5D theory.[5]

In terms of the 5D pure Einstein gravity, the rotating KK solution is written as

$$ds^2_{(5)} = \frac{H_2}{H_1}(R\,d\hat{y} + \boldsymbol{A})^2 - \frac{H_3}{H_2}(d\hat{t} + \boldsymbol{B})^2 + H_1\Big(\frac{d\hat{r}^2}{\Delta} + d\theta^2 + \frac{\Delta}{H_3}\sin^2\theta\, d\phi^2\Big), \tag{3.12}$$

in which

$$\begin{aligned} H_1 &= \hat{r}^2 + \mu^2 j^2\cos^2\theta + \hat{r}(p-2\mu) + \frac{1}{2}\frac{p}{p+q}(p-2\mu)(q-2\mu) \\ &\quad + \frac{1}{2}\frac{p}{p+q}\sqrt{(p^2-4\mu^2)(q^2-4\mu^2)}\,j\cos\theta, \end{aligned} \tag{3.13}$$

$$\begin{aligned} H_2 &= \hat{r}^2 + \mu^2 j^2\cos^2\theta + \hat{r}(q-2\mu) + \frac{1}{2}\frac{q}{p+q}(p-2\mu)(q-2\mu) \\ &\quad - \frac{1}{2}\frac{q}{p+q}\sqrt{(p^2-4\mu^2)(q^2-4\mu^2)}\,j\cos\theta, \end{aligned} \tag{3.14}$$

$$H_3 = \hat{r}^2 + \mu^2 j^2\cos^2\theta - 2\mu\hat{r}, \tag{3.15}$$

$$\Delta = \hat{r}^2 + \mu^2 j^2 - 2\mu\hat{r}, \tag{3.16}$$

$$\boldsymbol{A} = -\left[\sqrt{\frac{q(q^2-4\mu^2)}{p+q}}\Big(\hat{r} + \frac{p-2\mu}{2}\Big) - \frac{1}{2}\sqrt{\frac{q^3(p^2-4\mu^2)}{p+q}}j\cos\theta\right]H_2^{-1}d\hat{t}$$

[4] Precisely speaking, it corresponds to the Myers-Perry black hole on an orbifolded space $\mathbb{R}^{1,4}/\mathbb{Z}_{N_6}$, with N_6 an even integer. Details of the transformations are given in [34].

[5] The central charges and the entropy we will obtain do not depend on the Kaluza-Klein radius R. Therefore they would also be valid for very small R compared to the black hole radius, where the masses of the higher Kaluza-Klein modes become very large and the description by (3.11) is expected to be exact. However, for ζ^y_n (5.57), which relate massive KK modes, this limit $R \to 0$ is a little subtle.

$$
+\left[-\sqrt{\frac{p(p^2-4\mu^2)}{p+q}}(H_2+\mu^2 j^2\sin^2\theta)\cos\theta \right.
$$

$$
\left. +\frac{1}{2}\sqrt{\frac{p(q^2-4\mu^2)}{p+q}}\left\{p\hat{r}-\mu(p-2\mu)+\frac{q(p^2-4\mu^2)}{p+q}\right\}j\sin^2\theta\right]H_2^{-1}d\phi, \tag{3.17}
$$

$$
\boldsymbol{B}=\frac{1}{2}\sqrt{pq}\,\frac{(pq+4\mu^2)\hat{r}-\mu(p-2\mu)(q-2\mu)}{p+q}H_3^{-1}j\sin^2\theta\,d\phi, \tag{3.18}
$$

where $\hat{y}\sim\hat{y}+2\pi$ and R is the radius of the Kaluza-Klein circle at $\hat{r}\to\infty$.[6] After the Kaluza-Klein reduction along $\hat{y}$ direction, we obtain a 4D black hole of the form

$$
ds^2_{(4)}=-\frac{H_3}{\sqrt{H_1H_2}}(d\hat{t}+\boldsymbol{B})^2+\sqrt{H_1H_2}\Big(\frac{d\hat{r}^2}{\Delta}+d\theta^2+\frac{\Delta}{H_3}\sin^2\theta\,d\phi^2\Big), \tag{3.19}
$$

$$
e^{2\Phi}=R^2\sqrt{\frac{H_1}{H_2}}, \tag{3.20}
$$

$$
\boldsymbol{A}_{(4)}=\frac{1}{R}\boldsymbol{A}. \tag{3.21}
$$

The rotating KK solution has four parameters (μ,j,q,p), which correspond to four physical parameters of the reduced 4D black hole, that is, the ADM mass M, angular momentum J, electric charge Q and magnetic charge P. The explicit relations between these parameters are [31]:

$$
M=\frac{p+q}{4G_4}, \tag{3.22}
$$

$$
J=\frac{\sqrt{pq}(pq+4\mu^2)}{4G_4(p+q)}j, \tag{3.23}
$$

$$
Q=\frac{1}{2}\sqrt{\frac{q(q^2-4\mu^2)}{p+q}}, \tag{3.24}
$$

$$
P=\frac{1}{2}\sqrt{\frac{p(p^2-4\mu^2)}{p+q}}. \tag{3.25}
$$

Here we set $J,Q,P\geq 0$ for simplicity. The possible range of the parameters for regular solutions are

$$
0\leq 2\mu\leq q,p,\quad 0\leq j\leq 1, \tag{3.26}
$$

[6] We follow this form of the solution from [31] and checked that this indeed satisfies 5D Ricci flat condition. Note that there are some typos in [31].

and the black hole is extremal when we take $\mu \to 0$ with j fixed finite.[7] The outer/inner horizons are given by

$$r_\pm = \mu\big(1 \pm \sqrt{1-j^2}\big), \tag{3.27}$$

which lead to the Bekenstein-Hawking entropy

$$S_{BH} = \frac{\pi\sqrt{pq}}{2G_4\hbar}\left(\frac{pq+4\mu^2}{p+q}\sqrt{1-j^2} + 2\mu\right). \tag{3.28}$$

The Hawking temperature is

$$\beta_H = \frac{1}{T_H} = \frac{\pi\sqrt{pq}}{\mu\hbar}\left(\frac{pq+4\mu^2}{p+q} + \frac{2\mu}{\sqrt{1-j^2}}\right). \tag{3.29}$$

On the event horizon, the rotational velocity Ω_ϕ, the electric potential Φ_E and the magnetic potential Φ_M in the 4D theory are

$$\Omega_\phi = \frac{p+q}{\sqrt{pq}}\frac{2\mu j}{2\mu(p+q)+(pq+4\mu^2)\sqrt{1-j^2}}, \tag{3.30}$$

$$\Phi_E = \frac{\pi T_H}{2\mu G_4\hbar}\sqrt{\frac{p(q^2-\mu^2)}{p+q}}\left(p + \frac{2\mu}{\sqrt{1-j^2}}\right), \tag{3.31}$$

$$\Phi_M = \frac{\pi T_H}{2\mu G_4\hbar}\sqrt{\frac{q(p^2-\mu^2)}{p+q}}\left(q + \frac{2\mu}{\sqrt{1-j^2}}\right), \tag{3.32}$$

respectively. These physical quantities satisfy the first law of black hole thermodynamics:

$$dM = T_H dS + \Phi_E dQ + \Phi_M dP - \Omega_\phi dJ. \tag{3.33}$$

We also notice that, in terms of the 5D geometry, the potential Ω_y corresponding to the Kaluza-Klein momentum is

$$\Omega_y = \frac{2G_4}{R}\,\Phi_E, \tag{3.34}$$

at the horizon.

[7] We can take another extremal limit $j \to 1$ with μ fixed finite, which corresponds to the so-called "fast rotation" case. We will not consider this in this paper because the near horizon limit would be difficult to analyze.

Since we will focus on the extremal case in this paper, we show the explicit form of (3.13)-(3.18), (3.22)-(3.25) and (3.28) in that case here:

$$H_1 = \hat{r}^2 + p\hat{r} + \frac{1}{2}\frac{p^2 q}{p+q}(1 + j\cos\theta), \tag{3.35}$$

$$H_2 = \hat{r}^2 + q\hat{r} + \frac{1}{2}\frac{pq^2}{p+q}(1 - j\cos\theta), \tag{3.36}$$

$$H_3 = \Delta = \hat{r}^2, \tag{3.37}$$

$$\begin{aligned} \boldsymbol{A} &= -\frac{q^{3/2}}{\sqrt{p+q}}\left[\hat{r} + \frac{p}{2}(1 - j\cos\theta)\right] H_2^{-1} d\hat{t} \\ &\quad + \left[-\frac{p^{3/2}}{\sqrt{p+q}}\cos\theta + \frac{1}{2}\sqrt{\frac{pq^2}{p+q}}\left(p\hat{r} + \frac{p^2 q}{p+q}\right) H_2^{-1} j\sin^2\theta\right] d\phi, \end{aligned} \tag{3.38}$$

$$\boldsymbol{B} = \frac{1}{2}\frac{(pq)^{3/2}}{p+q}\frac{j\sin^2\theta}{\hat{r}}\, d\phi, \tag{3.39}$$

$$M = \frac{p+q}{4G_4}, \tag{3.40}$$

$$J = \frac{(pq)^{3/2}}{4G_4(p+q)}\, j, \tag{3.41}$$

$$Q = \frac{1}{2}\sqrt{\frac{q^3}{p+q}}, \tag{3.42}$$

$$P = \frac{1}{2}\sqrt{\frac{p^3}{p+q}}, \tag{3.43}$$

$$S_{BH} = \frac{\pi}{2G_4\hbar}\frac{(pq)^{3/2}}{p+q}\sqrt{1-j^2} = \frac{2\pi}{\hbar}\sqrt{\frac{P^2Q^2}{G_4^2} - J^2}. \tag{3.44}$$

Before closing this section, we rewrite the entropy calculated above in terms of the integer charges. As discussed in [34], the electric and the magnetic charges are quantized and written as

$$Q = \frac{2G_4\hbar N_0}{R}, \quad P = \frac{RN_6}{4}, \tag{3.45}$$

where N_6 and N_0 are integer numbers which corresponds to the number of D6-branes and D0-branes, respectively, if we embed the 5D KK black hole in IIA string theory with $R = g_s l_s$. In addition to this, J is also quantized as a result of the usual quantization of the angular momentum, so

$$J = \frac{\hbar N_J}{2}, \tag{3.46}$$

where N_J is an integer. By using these quantized quantities, the entropy (3.44) in the extremal case is also written as a quantized form:

$$S_{BH} = \pi\sqrt{N_0^2 N_6^2 - N_J^2}. \tag{3.47}$$

4 Near-Horizon Geometry of Extremal Rotating Kaluza-Klein Black Holes

Here we derive the near horizon geometry of the extremal ($\mu = 0$) rotating KK black holes. It is already investigated in [47, 48], and related discussions about symmetries of near-horizon geometries are given in [49].

We first introduce near horizon coordinates as

$$t = \lambda\hat{t}, \quad r = \frac{\hat{r}}{\lambda}, \quad y = \hat{y} - \frac{1}{R}\sqrt{\frac{p+q}{q}}\,\hat{t}, \tag{4.48}$$

while θ and ϕ are unchanged although the black holes are rotating along the ϕ direction. We will see that these coordinates are appropriate for obtaining the near-horizon geometry.

The near-horizon limit is defined as $\lambda \to 0$ in (4.48). In this limit, under the extremal condition $\mu = 0$, the metric (3.12) turns to

$$\begin{aligned} ds^2 =& \frac{q}{p}\frac{1 - j\cos\theta}{1 + j\cos\theta}\Big(Rdy + \frac{2r}{q(1 - j\cos\theta)}\sqrt{\frac{p+q}{q}}dt + \sqrt{\frac{p^3}{p+q}}\frac{j - \cos\theta}{1 - j\cos\theta}d\phi\Big)^2 \\ &- \frac{2(p+q)}{q^2 p(1 - j\cos\theta)}\Big(r\,dt + \frac{(pq)^{3/2}}{2(p+q)}j\sin^2\theta\,d\phi\Big)^2 \\ &+ \frac{p^2 q(1 + j\cos\theta)}{2(p+q)}\Big(\frac{dr^2}{r^2} + d\theta^2 + \sin^2\theta\,d\phi^2\Big), \end{aligned} \tag{4.49}$$

which we call *near-horizon extremal rotating Kaluza-Klein* black hole (NHERKK) geometry. This geometry is a so-called squashed $AdS_2 \times S^2$ with Kaluza-Klein $U(1)$ fibration on it.

We can also rewrite (4.49) by introducing

$$\rho = \frac{r}{2PQ}, \quad z = \frac{R}{2P}y \tag{4.50}$$

as

$$\begin{aligned} ds^2 = 2P^{4/3}Q^{2/3}\Bigg[&\frac{2(1 - j\cos\theta)}{1 + j\cos\theta}\Big(dz + \frac{\rho}{1 - j\cos\theta}dt + \frac{j - \cos\theta}{1 - j\cos\theta}\,d\phi\Big)^2 \\ &- \frac{1}{1 - j\cos\theta}\big(\rho\,dt + j\sin^2\theta\,d\phi\big)^2 + (1 + j\cos\theta)\Big(\frac{d\rho^2}{\rho^2} + d\theta^2 + \sin^2\theta\,d\phi^2\Big)\Bigg], \end{aligned} \tag{4.51}$$

where

$$z \sim z + 2\pi \frac{R}{2P}. \tag{4.52}$$

This metric is invariant under a transformation $t \to Ct, \rho \to \rho/C$ for an arbitrary constant C.

Both of these coordinates here are of Poincaré-type, and they do not cover the whole space in a single patch. Like the usual AdS_2 space, the whole NHERKK space is expected to have two disconnected boundaries. The boundary which is found in our coordinates is r (or ρ) $\to \infty$, which should be (a part of) one of the two. Therefore when we would like to focus on one of the dual chiral CFT_2's, which lives on one of the two boundaries, we can expect that our coordinates work well.

5 Boundary Conditions and Central Charges

In order to calculate the entropy of the rotating KK black holes, we have to determine the central charges of the Virasoro algebras which will act on the Hilbert spaces of the boundary theories.

5.1 Boundary Conditions and Asymptotic Symmetry Groups

By following the work by Brown and Henneaux [16], the first step is to find some boundary condition on the asymptotic variations of the metric and the ASG which preserves this boundary condition nontrivially. In fact, for the NHERKK metric (4.49), we can see that (at least) two different boundary conditions are allowed in order that some nontrivial ASG's exist.

5.1.1 two boundary conditions for the metric

Let us suppose that the metric is perturbed as $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ where $\bar{g}_{\mu\nu}$ is (4.49) and $h_{\mu\nu}$ is some deviation from it. One of the possible boundary conditions for $h_{\mu\nu}$ is,

$$\begin{pmatrix} h_{tt} = \mathcal{O}(r^2) & h_{tr} = \mathcal{O}(\frac{1}{r^2}) & h_{t\theta} = \mathcal{O}(\frac{1}{r}) & h_{t\phi} = \mathcal{O}(r) & h_{ty} = \mathcal{O}(1) \\ h_{rt} = h_{tr} & h_{rr} = \mathcal{O}(\frac{1}{r^3}) & h_{r\theta} = \mathcal{O}(\frac{1}{r^2}) & h_{r\phi} = \mathcal{O}(\frac{1}{r}) & h_{ry} = \mathcal{O}(\frac{1}{r}) \\ h_{\theta t} = h_{t\theta} & h_{\theta r} = h_{r\theta} & h_{\theta\theta} = \mathcal{O}(\frac{1}{r}) & h_{\theta\phi} = \mathcal{O}(\frac{1}{r}) & h_{\theta y} = \mathcal{O}(\frac{1}{r}) \\ h_{\phi t} = h_{t\phi} & h_{\phi r} = h_{r\phi} & h_{\phi\theta} = h_{\theta\phi} & h_{\phi\phi} = \mathcal{O}(\frac{1}{r}) & h_{\phi y} = \mathcal{O}(1) \\ h_{yt} = h_{ty} & h_{yr} = h_{ry} & h_{y\theta} = h_{\theta y} & h_{y\phi} = h_{\phi y} & h_{yy} = \mathcal{O}(1) \end{pmatrix}, \tag{5.53}$$

and a general diffeomorphism which preserves (5.53) is written as

$$\zeta = \Big[C_1 + \mathcal{O}\big(\frac{1}{r^3}\big)\Big]\partial_t + [-r\gamma'(y) + \mathcal{O}(1)]\partial_r + \mathcal{O}\big(\frac{1}{r}\big)\partial_\theta \\ + \Big[C_2 + \mathcal{O}\big(\frac{1}{r^2}\big)\Big]\partial_\phi + \Big[\gamma(y) + \mathcal{O}\big(\frac{1}{r^2}\big)\Big]\partial_y, \tag{5.54}$$

where C_1, C_2 are arbitrary constants and $\gamma(y)$ is an arbitrary function of y. From this, the asymptotic symmetry group is generated by the diffeomorphisms of the form[8]

$$\zeta^\phi = \partial_\phi, \tag{5.55}$$

$$\zeta^y_\gamma = \gamma(y)\partial_y - r\gamma'(y)\partial_r. \tag{5.56}$$

Especially, (5.56) generates the conformal group of the Kaluza-Klein circle. To see that it really obeys the Virasoro algebra, we expand $\gamma(y)$ in modes and define $\gamma_n = -e^{-iny}$. Then we can see that ζ^y_n, which are defined as

$$\zeta^y_n = \gamma_n\partial_y - r\gamma'_n\partial_r, \tag{5.57}$$

which obey the Virasoro algebra under the Lie bracket as

$$[\zeta^y_m,\ \zeta^y_n]_{Lie} = -i(m-n)\zeta^y_{m+n}. \tag{5.58}$$

We notice that the Virasoro generators are constructed from r and y. In other words, we see that the generators of the Virasoro algebra act on only y-direction in the dual boundary field theory. Thus it is very different from the usual holographic dual CFT_2 where the time direction t play some role. It seems that we cannot describe dynamical processes by using this Virasoro algebra, but at least to calculate the entropy, we can use the Virasoro algebra on the y-direction.

The other allowed boundary condition is,

$$\begin{pmatrix} h_{tt} = \mathcal{O}(r^2) & h_{tr} = \mathcal{O}(\frac{1}{r^2}) & h_{t\theta} = \mathcal{O}(\frac{1}{r}) & h_{t\phi} = \mathcal{O}(1) & h_{ty} = \mathcal{O}(r) \\ h_{rt} = h_{tr} & h_{rr} = \mathcal{O}(\frac{1}{r^3}) & h_{r\theta} = \mathcal{O}(\frac{1}{r^2}) & h_{r\phi} = \mathcal{O}(\frac{1}{r}) & h_{ry} = \mathcal{O}(\frac{1}{r}) \\ h_{\theta t} = h_{t\theta} & h_{\theta r} = h_{r\theta} & h_{\theta\theta} = \mathcal{O}(\frac{1}{r}) & h_{\theta\phi} = \mathcal{O}(\frac{1}{r}) & h_{\theta y} = \mathcal{O}(\frac{1}{r}) \\ h_{\phi t} = h_{t\phi} & h_{\phi r} = h_{r\phi} & h_{\phi\theta} = h_{\theta\phi} & h_{\phi\phi} = \mathcal{O}(1) & h_{\phi y} = \mathcal{O}(1) \\ h_{yt} = h_{ty} & h_{yr} = h_{ry} & h_{y\theta} = h_{\theta y} & h_{y\phi} = h_{\phi y} & h_{yy} = \mathcal{O}(\frac{1}{r}) \end{pmatrix}, \tag{5.59}$$

and general diffeomorphism preserving (5.59) can be written as

$$\zeta = \Big[C_1 + \mathcal{O}\big(\frac{1}{r^3}\big)\Big]\partial_t + [-r\epsilon'(\phi) + \mathcal{O}(1)]\partial_r + \mathcal{O}\big(\frac{1}{r}\big)\partial_\theta \\ + \Big[\epsilon(\phi) + \mathcal{O}\big(\frac{1}{r^2}\big)\Big]\partial_\phi + \Big[C_3 + \mathcal{O}\big(\frac{1}{r^2}\big)\Big]\partial_y, \tag{5.60}$$

[8] (5.54) also includes $\zeta^t = \partial_t$, but it is excluded from the ASG by requiring a constraint which we will explain in §5.1.2,

where C_1, C_3 are arbitrary constants and $\epsilon(\phi)$ is an arbitrary function of ϕ. The ASG is generated by

$$\zeta_\epsilon^\phi = \epsilon(\phi)\partial_\phi - r\epsilon'(\phi)\partial_r, \tag{5.61}$$

$$\zeta^y = \partial_y. \tag{5.62}$$

In exactly the similar manner as above, we define $\epsilon_n = -e^{-in\phi}$ and

$$\zeta_n^\phi = \epsilon_n\partial_\phi - r\epsilon_n'\partial_r, \tag{5.63}$$

which obey the Virasoro algebra

$$[\zeta_m^\phi,\ \zeta_n^\phi]_{Lie} = -i(m-n)\zeta_{m+n}^\phi. \tag{5.64}$$

In this case the Virasoro generator is constructed from r and ϕ.

Here we assume that these two boundary conditions correspond to two different realizations of these classical theories in a full quantum theory of gravity, like string theory. We will see that these boundary conditions indeed lead to the correct black hole entropy. This suggests that this very interesting phenomenon, i.e. two completely different microscopic theories for one geometry, may be true.

Note that in each of these boundary conditions, the Virasoro symmetry comes from an enhancement of one of the two $U(1)$ symmetries of the geometry, with the other remaining unenhanced. One may wonder whether both of the $U(1)$ symmetries could be enhanced at the same time with $\{\zeta_n^y\}$ and $\{\zeta_n^\phi\}$ living together in the ASG, leading to a dual CFT with two Virasoro symmetries. For example, we can consider a more relaxed boundary condition with $h_{t\phi} = \mathcal{O}(r)$, $h_{ty} = \mathcal{O}(r)$, $h_{\phi\phi} = \mathcal{O}(1)$, $h_{yy} = \mathcal{O}(1)$ and the other elements same as (5.53) and (5.59). This boundary condition indeed admits both $\{\zeta_n^y\}$ and $\{\zeta_n^\phi\}$. However in this case, the ASG includes a wider class of diffeomorphisms, some of which are not commutative with ζ^t. It means that we cannot fix the energy of the black hole, therefore this boundary condition cannot be regarded as consistent.[9]

5.1.2 the energy constraint

Using (2.5), the asymptotic conserved charges corresponding to ζ^t, (5.56) and (5.61) are defined by

$$Q_{\zeta^t} = \frac{1}{8\pi G_5}\int_{\partial\Sigma} k_{\zeta^t}, \quad Q_{\zeta_\gamma^y} = \frac{1}{8\pi G_5}\int_{\partial\Sigma} k_{\zeta_\gamma^y}, \quad Q_{\zeta_\epsilon^\phi} = \frac{1}{8\pi G_5}\int_{\partial\Sigma} k_{\zeta_\epsilon^\phi}, \tag{5.65}$$

[9] We thank Andrew Strominger, for suggesting our mistakes at this point in the original version of this paper.

where $G_5 = 2\pi R G_4$ is the 5D Newtonian constant and k_ζ is defined by (2.6). Obviously (5.55) and (5.62) are the cases of $\epsilon = 1$ in (5.61) and $\gamma = 1$ in (5.56) respectively, whose charges represent the variances of the angular momentum and the KK momentum. Similarly to the case of the 4D Kerr black hole, the generator ζ^t is also included in the asymptotic symmetry algebra. Therefore we set $Q_{\zeta^t} = 0$ identically in order that the black holes remain extremal.

5.2 Central Charges

Next we have to determine the centrally extended expressions of the Virasoro algebras. For the (r, y)-diffeomorphism (5.56), the second term on the right hand side of (2.7) is calculated as

$$\frac{1}{8\pi G_5} \int_{\partial\Sigma} k_{\zeta^y_m}[\mathcal{L}_{\zeta^y_n}\bar{g}, \bar{g}] = -i\frac{2}{G_4 R} Q\big(P^2 m^3 + \frac{R^2}{2} m\big)\delta_{m+n,0}. \tag{5.66}$$

From this, by defining the Virasoro operators L^y_m of y-direction as

$$\hbar L^y_m = Q_{\zeta^y_m} + \frac{1}{G_4 R} Q\Big(P^2 + \frac{R^2}{2}\Big)\,\delta_{m,0}, \tag{5.67}$$

replacing $\{\cdot,\ \cdot\}_{Dirac} \to \frac{1}{i\hbar}[\cdot,\ \cdot]$ and substituting into (2.7), we finally have the Virasoro algebra

$$[L^y_m,\ L^y_n] = (m-n)L^y_{m+n} + \frac{2}{G_4 R} Q P^2 (m^3 - m)\,\delta_{m+n,0}, \tag{5.68}$$

Therefore the central charge c^y is

$$c^y = \frac{24}{G_4 \hbar R} Q P^2 = 3 N_0 N_6^2. \tag{5.69}$$

For the (r, ϕ)-diffeomorphism (5.61), we can calculate the central charge in a similar manner. The second term on the right hand side of (2.7) is calculated as

$$\frac{1}{8\pi G_5} \int_{\partial\Sigma} k_{\zeta^\phi_m}[\mathcal{L}_{\zeta^\phi_n}\bar{g}, \bar{g}] = -iJ(m^3 - 2m)\delta_{m+n,0}. \tag{5.70}$$

Then by defining L^ϕ_m as

$$\hbar L^\phi_m = Q_{\zeta^\phi_m} - \frac{J}{2}\,\delta_{m,0}, \tag{5.71}$$

we can see that the Virasoro algebra is

$$[L^\phi_m,\ L^\phi_n] = (m-n)L^\phi_{m+n} + J\,(m^3 - m)\delta_{m+n,0}. \tag{5.72}$$

Therefore the central charge c^ϕ is

$$c^\phi = \frac{12}{\hbar} J = 6N_J. \tag{5.73}$$

Before closing this section, we note that the calculated central charges are integer numbers. In particular for (r, y) case, it is written by N_0 and N_6 only. This suggests that there is an underlying microscopic theory which is obtained from some weak coupling theory on D-branes. Since the central charge is completely different from the one obtained in [33, 34], it is highly interesting to investigate the corresponding D-brane system.

6 Temperatures

In the previous section, we derived the central charge of the Virasoro algebra for two cases. Next we have to determine the "temperature" T of the chiral CFT_2 following [26], since entropy is microscopically calculated by the thermal representation of the Cardy formula

$$S = \frac{\pi^2}{3} cT. \tag{6.74}$$

For this purpose, let us consider a free scalar field Ψ propagating on (3.12). It can be expanded as

$$\Psi = \sum_{\omega,k,m,l} e^{-i\omega\hat{t}+ik\hat{y}+im\hat{\phi}} f_{m,l}(r,\theta), \tag{6.75}$$

where ω is the asymptotic energy of the scalar field, while k and m are Kaluza-Klein momentum in the y-direction and the quantum number corresponding to the angular velocity respectively. We also notice that m, l label the spherical harmonics. Using the coordinates of the near-horizon geometry (4.48), we see that

$$e^{-i\omega\hat{t}+ik\hat{y}+im\hat{\phi}} = e^{-in^t t+in^y y+in^\phi \phi}, \tag{6.76}$$

$$n^t = \frac{1}{\lambda}\Big(\omega - \frac{k}{R}\sqrt{\frac{p+q}{q}}\Big), \quad n^y = k, \quad n^\phi = m. \tag{6.77}$$

After tracing out the states inside the horizon, we find that the vacuum state is expected to include a Boltzmann factor of the form

$$e^{-\frac{\hbar(\omega - k\Omega_y + m\Omega_\phi)}{T_H}} = e^{-\frac{n^t}{T^t}-\frac{n^y}{T^y}-\frac{n^\phi}{T^\phi}}. \tag{6.78}$$

The temperatures T^t, T^y and T^ϕ are calculated as

$$T^t = \frac{T_H}{\hbar\lambda}, \quad T^y = \frac{T_H}{\hbar\left(\frac{1}{R}\sqrt{\frac{p+q}{q}} - \Omega_y\right)}, \quad T^\phi = \frac{T_H}{\hbar\Omega_\phi}, \tag{6.79}$$

where we used (6.77). When $\mu, \lambda \to 0$ as $\mu/\lambda \to 0$, we see that $T^t \to 0$, while T^y and T^ϕ are

$$T^y = \frac{G_4 R}{4\pi P^2 Q}\sqrt{\frac{P^2Q^2}{G_4^2} - J^2} = \frac{1}{\pi N_0 N_6^2}\sqrt{N_0^2 N_6^2 - N_J^2}, \tag{6.80}$$

$$T^\phi = \frac{1}{2\pi J}\sqrt{\frac{P^2Q^2}{G_4^2} - J^2} = \frac{1}{2\pi N_J}\sqrt{N_0^2 N_6^2 - N_J^2}. \tag{6.81}$$

7 Microscopic Entropy

Using (6.74), either from (5.69) and (6.80) or from (5.73) and (6.81), we finally obtain the microscopic entropy as

$$\begin{aligned} S_{micro} &= \frac{\pi^2}{3} c^y T^y = \frac{\pi^2}{3} c^\phi T^\phi \\ &= \frac{2\pi}{\hbar}\sqrt{\frac{P^2Q^2}{G_4^2} - J^2} = \pi\sqrt{N_0^2 N_6^2 - N_J^2}, \end{aligned} \tag{7.82}$$

which exactly agrees with each other and with the one derived macroscopically, (3.44). Note that the Cardy formula will be valid for $T \gg 1$, which is satisfied when $N_0 N_6 \gg N_J$ for T^ϕ. For other cases, like the Kerr/CFT correspondence [26], the Cardy formula is not guaranteed to be applicable although we hope it is.

Usually the Cardy formula is written as $S = 2\pi\sqrt{\frac{cL_0}{6}}$. Therefore it is valuable to calculate the corresponding level of the Virasoro L_0. For (5.69) and (6.80), this can be written as

$$L_0^y = \frac{\pi^2}{6} c^y (T^y)^2 = \frac{R(P^2Q^2 - G_4^2 J^2)}{4G_4 \hbar P^2 Q} = \frac{N_0^2 N_6^2 - N_J^2}{2N_0 N_6^2}. \tag{7.83}$$

Similarly for (5.73) and (6.81), we obtain

$$L_0^\phi = \frac{\pi^2}{6} c^\phi (T^\phi)^2 = \frac{P^2Q^2 - G_4^2 J^2}{2G_4^2 \hbar J} = \frac{N_0^2 N_6^2 - N_J^2}{4N_J}. \tag{7.84}$$

8 Conclusion and Discussion

In this paper, we calculated the entropy of the extremal rotating Kaluza-Klein black holes microscopically by using Brown-Henneaux's method. Following [26], we imposed appropriate boundary conditions on the near horizon geometry of the black holes and then identified the diffeomorphisms which preserve the boundary conditions with the generators of the Virasoro algebras. Then by calculating the Dirac brackets of the corresponding conserved charges, we determined the Virasoro algebras with non-vanishing central charges. At the same time, we relate the physical parameters of the black holes with the quantum numbers on the near horizon geometry. Then we determined the temperatures of the dual chiral CFT_2's. From the central charges and the temperatures, by using the Cardy formula, we calculated the entropy of the extremal rotating Kaluza-Klein black holes microscopically, which agrees with the one obtained macroscopically.

The rotating Kaluza-Klein black holes are known to be related to spinning D0-D6 bound states and we can also calculate the entropy from the D-brane viewpoint. Therefore we expect that we can obtain a deeper understanding of the chiral CFT_2's by considering a relation between our calculation and that by using the D-brane configuration.

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Note added

As this article was being completed, we received the preprint [50]. In that paper, the Kerr/CFT correspondence is applied to the higher dimensional Myers-Perry black holes and the Kerr-AdS black holes.

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